Harmonics In A SVPWM Scheme For V/F Control Of An Induction Motor

Abstract

In this paper a V/f control using a simple space vector modulation scheme is suggested. Basically a rotating space vector is required. This is achieved by generating space vectors in each sector of the space vector hexagon one after another after a switching time interval. Intuition tells us that more the number of space vectors in each sector, lesser the harmonics generated and smoother the output waveform. This paper examines the harmonics resulting from a simple SVPWM algorithm used to do V/f control of an induction motor and implemented using the TMS320F28335 DSP.

1. Introduction

Speed control of an induction motor can be achieved today by using V/f control for coarse control or accurate control using vector control or DTC (Direct torque control)[1-2]. In this paper an inexpensive V/f control algorithm is put forward which uses voltage space vectors to generate a three phase voltage supply to run an induction motor [3]. The harmonics present in the phase voltages are measured, and help analyse the efficacy of the system [4]. A number of voltage space vectors are generated in each sector of the space vector hexagon, the resulting phase voltages are observed and their Fourier analysis done. In our approach the induction motor speed taken in electrical Hertz is divided into four ranges. The number of vectors per sector to be generated depends on the speed range. Lower the speed; more are the space vectors to be generated [5-6]. One simple strategy tried was to use one space vector per sector when the motor speed is 45 to 50 Hz. Two space vectors per sector when the speed is between 30-45 Hz, three space vectors per sector when the speed is between 15-30Hz and four below 15Hz. The harmonic analysis was done for this in order to formulate a better strategy if required. The space vectors are generated in a
traditional pattern, the dwell a time are stored in tables, as for a V/f control, the dwell times for a speed range are constant and only the dead time to, and varies with the frequency.

Figure 1: Space Vector in a sector

![Space Vector in a sector](image)

\[
T_s = \frac{V_1}{V_{dc}} \cdot T_s \cdot \frac{\sin(60^\circ - \theta)}{\sin(60^\circ)} \rightarrow (1)
\]

\[
T_2 = \frac{V_1}{V_{dc}} \cdot T_s \cdot \frac{\sin(\theta)}{\sin(60^\circ)} \rightarrow (2)
\]

\[
T_0 = T_S - T_1 - T_2 \rightarrow (3)
\]

Ts= sampling time, To= zero vector time duration, T1 and T2 are time durations of two adjacent vectors. At 50 Hz, the phase voltage applied on the motor needs to be the rated voltage of amplitude 325V per phase. Hence V1 needs to be \((3/2)(325)=487.5V\). Vdc*Cos(30°)=487.5V orVdc=562.9 Volts.

Hence the DC link voltage of the voltage source inverter is maintained at 563V constant. Thus at 50Hz operation, \(T_S= (1/6)(1/50)=3.33\)ms, \(T_1 = 3.33\)ms, \(T_2 = 0, T_0=0, \) (Using the above three equations (1),(2),and(3)).

**Table -1: Dwell times around 50Hz**

<table>
<thead>
<tr>
<th>45 Hz&lt;f&lt;=50 Hz, (\theta=0)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(T_S)</td>
</tr>
<tr>
<td>(T_1)</td>
</tr>
<tr>
<td>(T_2)</td>
</tr>
<tr>
<td>(T_0)</td>
</tr>
</tbody>
</table>

Now consider the frequency range from 30 to 45 Hz, where two space vectors are produced per sector. The angles are \(\theta=0\) and \(\theta=30^\circ\). Ts, To, T1 and T2 are calculated while maintaining constant V/f.
Table-2: Two vectors per sector

<table>
<thead>
<tr>
<th>30 Hz&lt;f&lt;=45Hz</th>
<th>(\theta=0^\circ)</th>
<th>(\theta=30^\circ)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(T_S)</td>
<td>(1/(12^*f))</td>
<td>(1/(12^*f))</td>
</tr>
<tr>
<td>(T_1)</td>
<td>1.443ms</td>
<td>0.833ms</td>
</tr>
<tr>
<td>(T_2)</td>
<td>0</td>
<td>0.833ms</td>
</tr>
<tr>
<td>(T_0)</td>
<td>(T_S-T_1-T_2)</td>
<td>(T_S-T_1-T_2)</td>
</tr>
</tbody>
</table>

Similar calculations are done for the frequency range from 15 to 30 Hz but with three space vectors in each sector which are tabulated as below:

Table-3: Three vectors per sector

<table>
<thead>
<tr>
<th>15 Hz&lt;f&lt;=30 Hz</th>
<th>(\theta=0^\circ)</th>
<th>(\theta=20^\circ)</th>
<th>(\theta=40^\circ)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(T_S)</td>
<td>(1/(18^*f))</td>
<td>(1/(18^*f))</td>
<td>(1/(18^*f))</td>
</tr>
<tr>
<td>(T_1)</td>
<td>0.954ms</td>
<td>0.708ms</td>
<td>0.376ms</td>
</tr>
<tr>
<td>(T_2)</td>
<td>0</td>
<td>0.376ms</td>
<td>0.708ms</td>
</tr>
<tr>
<td>(T_0)</td>
<td>(T_S-T_1-T_2)</td>
<td>(T_S-T_1-T_2)</td>
<td>(T_S-T_1-T_2)</td>
</tr>
</tbody>
</table>

It may be seen that in table-3, the dwell times at \(\theta=20^\circ\) and for \(\theta=40^\circ\) get swapped, which is explained by the symmetry of the two vectors. Finally for frequency below 15Hz, four vectors per sector are used. The table below gives the dwell times for the vectors.

Table-4: Four vectors per sector

<table>
<thead>
<tr>
<th>0 Hz&lt;f&lt;=15 Hz</th>
<th>(\theta=0^\circ)</th>
<th>(\theta=15^\circ)</th>
<th>(\theta=30^\circ)</th>
<th>(\theta=45^\circ)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(T_S)</td>
<td>(1/(24^*f))</td>
<td>(1/(24^*f))</td>
<td>(1/(24^*f))</td>
<td>(1/(24^*f))</td>
</tr>
<tr>
<td>(T_1)</td>
<td>0.715ms</td>
<td>0.584ms</td>
<td>0.413ms</td>
<td>0.214ms</td>
</tr>
<tr>
<td>(T_2)</td>
<td>0</td>
<td>0.214ms</td>
<td>0.413ms</td>
<td>0.584ms</td>
</tr>
<tr>
<td>(T_0)</td>
<td>(T_S-T_1-T_2)</td>
<td>(T_S-T_1-T_2)</td>
<td>(T_S-T_1-T_2)</td>
<td>(T_S-T_1-T_2)</td>
</tr>
</tbody>
</table>

The PWM pattern can be synthesized easily using these tables alone and the code is simplified. The code can be written to include a gradual speed variation from the present speed to the required motor speed. Space vector \(V_1\) in a sector is shown in fig (1). it is time averaged of two adjacent vectors in a sector.

2. Implementation Of SVPWM

In a sampling interval of \(T_S\) the durations of \(T_1\) and \(T_2\) have to be defined. The dead time \(T_0\) is split into two halves, first a delay of \(T_0/2\) is given after which \(T_1\) and \(T_2\) are constructed followed by the remaining \(T_0/2\). This is done by the “compare” mode in a timer of the DSP. The DSP has an up-counting counter with a time period of \(T_S\). The up-counting timer may be represented as a triangular saw tooth waveform. The timer’s output is high initially while the value in the compare register is being compared to the up counting timer register. When there is a match the output goes low. The value of \(T_0/2\) is compared first. Simultaneously the value of \(T_0+T_1\) is compared and so also
To/2+ T₁+ T₂. By subtraction of the levels of the signals as shown in the figure 5, we can obtain the durations T₁ and T₂.

In fig. 2, are shown the three “compare” outputs P, Q and R which are the PWM outputs of the DSP. “Q-P” gives the T₁ duration and “R-Q” gives the T₂ duration. Digitally they may be obtained by “EX-ORing” instead of subtraction.

The sector in which the vector lies is given by the sector number “SECTOR” which consists of three bits, like 001 for the first sector or 010 for the second etc. The SECTOR determines whether the switch gate pulse is produced in the T₁ interval or the T₂ period so that the overall space vector is averaged.

Table 5: Switch selection table

<table>
<thead>
<tr>
<th>Sl. No.</th>
<th>SECTOR</th>
<th>Sa₁</th>
<th>Sa₂</th>
<th>Sb₁</th>
<th>Sb₂</th>
<th>Sc₁</th>
<th>Sc₂</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>001</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>010</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>011</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>100</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>5</td>
<td>101</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>6</td>
<td>110</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>7</td>
<td>Others</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

The table is formed by an inspection of the six vectors forming the radii of the space vector hexagon. The gate pulses Sa, Sb and Sc to the upper three switches of the three phases are found as below, the bottom switches are complements of the top with dead time added.

Sa = T₁.Sa₁+ T₂. Sa₂

Sb = T₁.Sb₁+ T₂. Sb₂

Sc = T₁.Sc₁+ T₂. Sc₂

Figure 2: Synthesis of pulses
3. Simulation Of SVPWM

The input to the simulation program is the frequency of operation. This determines the number of vectors to be constructed per sector and a triangular carrier of the required frequency is generated. Next an appropriate look up table is used to find the levels \( T_0/2, T_0/2+T_1 \) and \( T_0/2+T_1+T_2 \) which are compared with the triangular carrier to produce \( P, Q \) and \( R \) as in Fig. 2. From this the pulses of duration \( T_1 \) and \( T_2 \) result. From the knowledge of the sector number, the gate pulses to the MOSFETs or IGBTs can be generated based on the switch selection table.

The simulation program was run and a harmonic analysis was done to see the relative harmonic levels in the line to line voltages at different frequencies of operation. The \( V/f \) ratio was also checked.

![Figure 3: Line to line voltage at 50 Hz.](image)

![Figure 4: Harmonic content in line to line voltage at 50 Hz](image)

![Figure 5: Line to line voltage at 45 Hz](image)
4. Observations

Line voltages and harmonics at different frequencies are shown in figure(1) to figure(10). At 50 Hz, the 6n±1 harmonics are seen and they decay rapidly from a value of 20% to a negligible value by 41st to 43rd order.

At 45 Hz, the 13th harmonic is 30% and the harmonics alternate in value while reducing to less than 5% by the 59th and 61st orders. At 30 Hz, the 17th and 19th components are prominent with magnitudes of 40% and 70% respectively. However by the 61st order the magnitudes die down to 10%. In the case of operation at 15 Hz, the magnitude of the 23rd harmonic is about 70% and the 25th harmonic is 101%. The 47th and 49th values are again dominant but other harmonics are small. The 25th harmonic corresponds to a frequency of 25x15=375 Hz which will see a higher inductive reactance which attenuates the current of this frequency.
A.N. Maheswarappa, Dr. Sanjay Lakshminarayanan: Harmonics In A SVPWM Scheme For V/F Control Of An Induction Motor
5. Motor Current
An induction motor model with following parameters was used to observe the output of the inverter.

\[ M = 0.272 \, \text{H} \]
\[ R_s = 2.08 \, \Omega \]
\[ R_r = 1.19 \, \Omega \]
\[ L_r = 0.28 \, \text{H} \]
\[ L_s = 0.28 \, \text{H} \]
\[ p = 2 \, \text{Pole pairs} \]
\[ B = 0.1 \]
\[ J = 0.25 \]

(All in SI units)

The motor currents and harmonics at different frequencies are shown in figure (11) to figure (16).

At 50Hz it has prominent 5th and 7th harmonics and a bit of 11th and 13th harmonics. At 30 Hz, the motor current has the harmonics at 17th and 19th predominate as 3 space vectors are generated in each sector; hence the \( 6n\pm 1 \) harmonics are active. The third harmonic is absent in all the cases.

At 15 Hz, the current has 23rd and 25th harmonics of amplitude 45% and 55% respectively.

![Figure 11: Motor phase current at 50Hz](image)

![Figure 12: Harmonics in the current at 50Hz.](image)
The motor speed seems to be close to 120f/P rpm with a slight amount of slip of about 5%.

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Figure 13: Motor current at 30 Hz

Figure 14: Harmonics in current at 30 Hz

Figure 15: Motor current waveform at 15 Hz
6. Conclusion
This method of SVPWM seems to work well with tolerable values of voltage and current harmonics and the motor speed responds to the excitation frequency quite well. This is achieved with simple code using DSP.

7. References
Author’s Biographies

Mr. A. N. Maheswarappa received BE degree in Electrical and Electronics Engineering from Mysore University, Karnataka, India in the year 1987 and M.Tech. Degree from NITK Surathkal, Karnataka, India in 1994. He is currently working at Global Academy of Technology in E&E Dept., Bangalore, India. He is having 27 years of teaching experience. At present he is pursuing towards his doctoral degree from VTU, Karnataka, India.

Dr Sanjay Lakshminarayanan received the B.Tech. degree from the Indian Institute of Technology (IIT), Kharagpur, India, in 1990 and the M.Sc. (Engg.) degree in Electrical Engineering from the Indian Institute of Science (IISc) Bangalore, India, in 1995 and Ph.D. degree in 2007 from the Indian Institute of Science (IISc), Bangalore, India. He has been in the industry for about ten years. He was with Grentel Technologies, Cochin, Hical Magnetics Pvt. Ltd, Bangalore, and GE Medical Systems, Bangalore and has a teaching experience of about Nine years. His research interests are in the area of power converters, PWM strategies, and motor drives.