## Abstract

This paper considers the trajectory tracking control of linear inverted pendulum (IP) system. The controller is designed based on the mathematical model of the inverted pendulum system. Non-linear IP is a multivariable system having angle of pendulum and position of cart are two variables to be controlled. For this purpose we used fuzzy logic controller to tune the PID gain and compare its response with conventional controller. Through simulation in Matlab by selecting appropriate fuzzy rules are designed to tune the parameters $k_p, k_i$, and $k_d$ of the PID controller, the performance of the inverted pendulum system has improved significantly compare to conventional PID controller.

## 1. Introduction

Inverted pendulum is one of the most difficult systems to control in the field of control engineering, because it is a non-linear as well as unstable system [1]. It provide a platform to test various control techniques and used to simulate experiments such as walking robots, missile guidance and flying objects in space etc. Here the aim of the study is to design a control system that keeps the pendulum
balanced and tracks the cart to a commanded position. The conventional PID controller is still used in industries, because of its simple in control structure, not too expensive and effective for a linear system [2]-[4]. The conventional PID controller can be used for virtually any process condition [5]-[7]. If an accurate mathematical model of a system is available to us then we can design a conventional PID controller. But the real world is not linear as well as uncertain and contains always incomplete data. So we cannot use the conventional PID controller if the system has a high level of complexity such as high order modeling non linearity, time delay and structural uncertainties [8].

Fuzzy is always best to handle the real world or imprecise data, so we are combine the conventional controllers with fuzzy logic controllers. The fuzzy logic controllers (FLC) have the ability to control a system by using some limited expert knowledge. The structures of the FLC consist of fuzzifier, rule base, fuzzy interference engine and a defuzzifier. In this paper, Self Tuning Fuzzy PID controller is developed to improve the performance of the inverted pendulum on a cart system [9]-[10]. The controller is designed based on the mathematical model of the system. Fuzzy logic is used to tune each parameters $k_p$, $k_i$ and $k_d$ of PID controller. The performance of the systems such as rise time, overshoot, settling time and error steady state can be improved by Self-Tuning Fuzzy PID controller [11]-[12]. In fuzzy sets the membership functions are certain or crisp, here the input variables in fuzzy system are mapped by a set of membership functions. A FLS is characterized by IF-THEN rules, its antecedent and consequent sets are fuzzy set [13]-[14]. A digital computer of the stored program concept was created to perform a variety of tasks in sequence. The operation can be easily changed by changing the program [15].

Through simulation in Matlab by selecting appropriate fuzzy rules are designed to tune the parameters $k_p$, $k_i$ and $k_d$ of the PID controller. In this study, we propose two controllers conventional PID and Self Tuning Fuzzy PID controller to balance the pendulum at upright position and analysis the result. We find the improvement in system performance over the conventional PID controller by the influence of the external disturbance. This paper is organized as follows: section II presents the analytic study of conventional PID and Self Tuning Fuzzy PID controller. In section III, the description of the mathematical model of the nonlinear inverted pendulum system and design of self-tuning fuzzy PID controller is contained. Finally, section IV presents the simulation results followed by the conclusion and the references.

### 2. Analytic Study Of Controllers

The main objectives of controller design are as follows: (1) To stabilize the pendulum at its upright position, (2) To uphold the cart position at the origin, (3) Tracking of desired position by pendulum cart, (4) To use minimum control effort required to control the pendulum angle and cart position.

#### A. Conventional Controller

PID (Proportional, Integral and Differential) controller is the most common form of feedback. In PID controller the basic idea is the examination of signals from sensors placed in the system, called feedback signals. Let’s consider Fig. 1 the below given unity feedback system,

The PID controller is usually implemented as follows:

\[ u(t) = k_p e(t) + k_i \int e(t) \, dt + k_d \frac{de(t)}{dt} \]  
\[ e(t) = y_p(t) - y_m(t) \]

Figure 1: System with PID controller
Where $k_p$, $k_i$, and $k_d$ are the proportional, the integral, and the derivative gains respectively. The controller output, the process output, and the set point are denoted as $u(t)$, $y_r(t)$ and $y_m(t)$ respectively. A proportional controller ($k_p$) will have effect of reducing the rise time, but never eliminates the steady-state error. An integral controller ($k_i$) will reduce the steady-state error but may make the transient response worse. A derivative controller ($k_d$) will have an effect on stability of the system, it reduces the overshoot, and improving the transient response. Effects of these three controllers can be summarized as shown in the Table 1.

<table>
<thead>
<tr>
<th>Gain parameters</th>
<th>Effect of increasing gain</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k_p$</td>
<td>Decrease, Increase, Small change, Decrease</td>
</tr>
<tr>
<td>$k_i$</td>
<td>Decrease, Increase, Increase, Eliminate</td>
</tr>
<tr>
<td>$k_d$</td>
<td>Small Change, Decrease, Decrease, Small change</td>
</tr>
</tbody>
</table>

To build a self-balancing pendulum on a cart we first derived the system equation then check its real time response (both time and frequency). Then we designed a PID controller to control the close loop function. The easiest way to tune a PID controller is to auto-tune the $P$, $I$ and $D$ parameters one at a time. The obtained final values are $k_p = 796.64$, $k_i = 919.05$, and $k_d = 62.64$

B. Self-Tuning Fuzzy PID Controller

The performance specifications of the systems such as rise time, overshoot, settling time and error steady state can be improved by tuning parameters $k_p$, $k_i$, and $k_d$ of the PID controller. By developing self tuning fuzzy controllers, these parameters can be modified online, according to the changes in the process condition without much intervention of an operator.

Lotfi Zadeh, the father of fuzzy logic is extend two valued logic, defined by the binary pair {0,1}, to the whole continuous interval [0, 1]. Fuzzy controllers use heuristic information in developing design the control of nonlinear dynamic system. A fuzzy control system is shown in Fig. 2. FLS is consist fuzzifier, rules, inference engine and output processor (defuzzifier) and that are interconnected. The fuzzifier converts the crisp value into Fuzzy Sets. It is needed to activate rules that are in terms of linguistic variables. The rules are the heart of an FLS. The rules are expressed as a collection of IF-THEN statements. The IF-part of a rule represents antecedent and the THEN part represents consequent. The fuzzified inputs activate the inference engine and the rule base to produce a Fuzzy Set output. The commonly used inferential procedure is minimum and maximum implication method. Defuzzification is necessary to obtain the crisp number as the output.

Here we used self-tuning fuzzy PID and PID controller, that is, the three parameters such as proportional gain ($k_p$), integral gain ($k_i$) and derivative gain ($k_d$) of controllers are tuned by using fuzzy tuner. The co-efficient of the classical controllers cannot be properly tuned for the non linear inverted pendulum system with unpredictable parameter variation, hence tune automatically the controller parameters such as $k_p$, $k_i$, $k_d$ values by using self-tuning fuzzy PID controller. The structure of the self-tuning fuzzy PID controller is shown in Fig. 3.
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3. Model Of The System

A. Mathematical Model of Inverted Pendulum

The system consists of mathematical model of inverted pendulum and model of self-tuning fuzzy PID controller. Model of the pendulum and controller was created in Matlab-Simulink program. The inverted pendulum is mounted on a moving cart is shown in the Fig. 4. A servomotor is controlling the translation motion of the cart, through a belt/pulley mechanism. A rotary potentiometer is used to feedback the angular motion of the pendulum to servo electronics to generate actuating signal. The controller circuits provide the controlling signal which then drives the cart through the servomotor and driving pulley/belt mechanism. To and fro motion of the cart applies moments on the inverted pendulum and keeps the pendulum upright. The Free Body Diagram of the system is used to obtain the equations of motion. Below given Fig. 5 shows the free body diagrams. The meaning and values of parameters are listed in Table 2

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cart mass ((m_c))</td>
<td>1.096 kg</td>
</tr>
<tr>
<td>Pendulum mass ((m_p))</td>
<td>0.109 kg</td>
</tr>
<tr>
<td>Friction co-efficient of cart ((b))</td>
<td>0.1 N/m/s</td>
</tr>
<tr>
<td>Distance from pendulum rotation axis centre to pendulum mass centre ((l))</td>
<td>0.25 m</td>
</tr>
<tr>
<td>Gravity acceleration ((g))</td>
<td>9.8 m/s²</td>
</tr>
<tr>
<td>Pendulum inertia ((I))</td>
<td>0.0034 kg.m²</td>
</tr>
</tbody>
</table>

Figure 2: A fuzzy control system

Figure 3: The structure of the self-tuning fuzzy PID controller

The proposed controller structures consist of a simple upper level controller and a lower level classical controller. The upper level controller provides a mechanism to select the gain of a classical PID and the lower level deliver the solution to a particular situation. Here we use the control structure as a rule based Mamdani fuzzy controller. It is used in the upper level and conventional PID controller is selected for the lower level.
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The variables are, $F$- Force acting on the cart, $x$- Cart position, $\phi$- Angle between the pendulum and vertically upward direction and $\theta$- Angle between the pendulum and vertically downward direction. To obtain the transfer function of the linearized system equation analytically, we must first take the Laplace transform of the system eqn. (3) to eqn. (4) [16-18].

\[(m_p + m_c)\ddot{x} + b\dot{x} - m_p l \dot{\phi} = u\]  
\[(I + m_p l^2)\ddot{\phi} - m_p gl \phi = m_p l \ddot{x}\]  

The obtained Transfer function is as:

\[
\frac{\phi(S)}{U(S)} = \frac{\frac{m_p l S}{q}}{S^3 + \frac{b(l+m_p l^2)}{q} S^2 \frac{m_p g l (m_c + m_p)}{q} \frac{b m_p gl}{q}} \]  

Where $q = \left( m_c + m_p \right) \left( l + m_p l^2 \right) - (m_p l)^2$  

\[
\frac{\phi(S)}{U(S)} = \frac{\frac{m_p l S}{q S^3 + b(l+m_p l^2) S^2 - m_p g l (m_c + m_p) S - b m_p g l}}{2.357 s} \]  

The obtained Transfer Function in term of values of parameters is as:

\[
\frac{\phi(S)}{U(S)} = \frac{\frac{m_p l S}{q S^3 + 0.08832 S^2 - 27.83 S - 2.309}}{2.357 s} \]

B. Design of Self Tuning Fuzzy PID Controller

In fuzzy structure, there are two inputs to fuzzy inference: error $e(t)$ and change of error $de(t)$ and three outputs for each PID controller parameters respectively $k_p, k_i$, and $k_d$. Fuzzy inference block of the controller design is shown in Fig. 6 below. The steps for designing aimed controller for the inverted pendulum are as follows:
a) Select the input and output parameters for the fuzzy controller. Here we choose the error signal and the change of error signal as the input parameters and output parameters for the fuzzy controller as the proportional, the integral, and the derivative gains.

![Fuzzy inference block of the controller](image)

**Figure 6: Fuzzy inference block of the controller**

b) Then divide the universe of discourse into FSs. Fig. 7 and 8 show the input membership functions for the error signal and change of error signal respectively. Here the universe of discourse is divided as Negative Large (NL), Negative Small (NS), Zero (Z), Positive Small (PS) and Positive Large (PL). Fig. 9 to 11 shows the output membership function for the proportional, the integral and derivative gains, whereas the universe of discourses is divided as Medium (M), Big (B) and Very Big (VB).

c) Write the rule base for the Self-Tuning Fuzzy PID controller, based on experience and it is described in the below given Table 3-5 correspondingly.

d) Use the algorithm of the aimed controller: Centroid defuzzification is the best technique to obtain the crisp output.
The degree of each membership function which was computed in the previous step of fuzzifications encountered by the subprogram called defuzzify and this after certain process it returns defuzzified output.

![Membership functions for the error signal](image)

**Figure 7: Membership functions for the error signal**

![Membership functions for the change of error signal](image)

**Figure 8: Membership functions for the change of error signal**
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Figure 9: Membership functions for the proportional gain

Figure 10: Membership functions for the integral gain

Figure 11: Membership functions for the derivative gain

Table 3: Rule Base For The Proportional Gain

<table>
<thead>
<tr>
<th>$\Delta e$</th>
<th>e</th>
<th>NL</th>
<th>NS</th>
<th>Z</th>
<th>PS</th>
<th>PL</th>
</tr>
</thead>
<tbody>
<tr>
<td>NL</td>
<td>M</td>
<td>S</td>
<td>S</td>
<td>S</td>
<td>S</td>
<td>M</td>
</tr>
<tr>
<td>NS</td>
<td>B</td>
<td>M</td>
<td>S</td>
<td>M</td>
<td>B</td>
<td>M</td>
</tr>
<tr>
<td>Z</td>
<td>VB</td>
<td>B</td>
<td>M</td>
<td>B</td>
<td>VB</td>
<td></td>
</tr>
<tr>
<td>PS</td>
<td>B</td>
<td>M</td>
<td>S</td>
<td>M</td>
<td>B</td>
<td></td>
</tr>
<tr>
<td>PL</td>
<td>M</td>
<td>S</td>
<td>S</td>
<td>S</td>
<td>M</td>
<td></td>
</tr>
</tbody>
</table>
4. Simulation Results

Here we compare the results of Self Tuning Fuzzy PID controller and PID controllers. The parameters of an inverted pendulum on a cart system are given in Table 2. Two different tasks are described below.

A. Closed loop response

Fig. 12 shows the response of input in closed loop for the initial condition $\gamma_m = 1$ rad and $\gamma_r = 0$ rad/sec. The controllers output are moved to the set point value without steady state error. Self-tuning Fuzzy PID controller takes less settling time, better set-point tracking, less overshoot and thereby producing minimum integral absolute error. The performance measures are tabulated in Table 6.

B. Regulatory response

The Fig. 13 shows the response of the inverted pendulum system using PID and self tuning Fuzzy PID controllers. Here we add the angle disturbance value as $d= 1$ rad after 0.7 sec from starting the simulation. In the presence of disturbances, the self tuning Fuzzy PID controller still outperforms PID...
controller and in self-tuning Fuzzy PID case less control effort is required. The self-tuning Fuzzy PID controller is made significantly better than the PID controller.

![Graph showing comparison between PID and self-tuning Fuzzy PID controllers]

Figure 13: The response of the inverted pendulum system when the value of disturbance value is d= 1 rad

To show the visual indications of the control performance, an objective measure of an error performance was made using the Integral of Square of Errors (ISE), the Integral Time-weighted Absolute Error (ITAE) and Integral Absolute Error (IAE) criteria. They are set in eqn. (10) to eqn. (12) given below

\[ ISE = \int_0^\infty [e(t)]^2 \, dt \quad (10) \]
\[ ITAE = \int_0^\infty t \, |e(t)| \, dt \quad (11) \]
\[ IAE = \int_0^\infty |e(t)| \, dt \quad (12) \]

The Table 6 list out the the Integral of Square of Errors (ISE), the Integral Time-weighted Absolute Error (ITAE) and Integral Absolute Error (IAE) values respectively for the PID and in self-tuning Fuzzy PID controllers for all the above experimental tasks. From the table we can conclude that the values of the ISE, ITAE and IAE for the proposed self-tuning Fuzzy PID controller are lower than that of the value obtained for the PID controller. That is, the response of the self-tuning Fuzzy PID controller is faster and superior to respond the uncertainties than the PID controller.

<table>
<thead>
<tr>
<th>Performance Indices</th>
<th>Controller</th>
<th>Closed Loop Response</th>
<th>Regulatory Response</th>
</tr>
</thead>
<tbody>
<tr>
<td>ISE</td>
<td>PID</td>
<td>0.005309</td>
<td>4.0432</td>
</tr>
<tr>
<td></td>
<td>self-tuning Fuzzy PID</td>
<td>0.009215</td>
<td>2.713</td>
</tr>
<tr>
<td>ITAE</td>
<td>PID</td>
<td>0.02361</td>
<td>0.0821</td>
</tr>
<tr>
<td></td>
<td>self-tuning Fuzzy PID</td>
<td>0.01773</td>
<td>0.0672</td>
</tr>
<tr>
<td>IAE</td>
<td>PID</td>
<td>0.03049</td>
<td>21.71</td>
</tr>
<tr>
<td></td>
<td>self-tuning Fuzzy PID</td>
<td>0.02893</td>
<td>15.68</td>
</tr>
</tbody>
</table>

5. Conclusion

In this paper, we design two kinds of controllers (i.e. PID and self-tuning Fuzzy PID controller) and analyse the responses from it by using MATLAB simulation to track the inverted pendulum system at commanded position. The result from the figures show that the performance of using PID and
self-tuning Fuzzy PID controllers, when closed loop case and regulatory response case is presented in the system. In closed loop response case, the controllers output are moved to the set point value without steady state error. Self-tuning Fuzzy PID controller takes less settling time, better set-point tracking, less overshoot and thereby producing minimum integral absolute error. In the presence of disturbances (regulatory response case), the self-tuning Fuzzy PID controller still outperforms PID controller and in self-tuning Fuzzy PID case less control effort is required. Hence, the fuzzy controller capable of handle the plant with system external disturbances. On the other hand, the self-tuning Fuzzy PID controller has a comparatively uniformed performance than the classical PID controller, no matter how large the external disturbance is presented in the system. Thus the self-tuning Fuzzy PID controller can be considered more robust.

6. References