Abstract

Vedic Mathematics provides methods of solutions of different types of equations such as algebraic equations, simultaneous linear equations, transcendental equations as well as differential equations. For solution of equations of various types the Vedic Sub-sutras give us some first principles which are known to the modern mathematicians but which are not actually used in practice as basic and fundamental first principles of a practically axiomatic character in mathematical computations. Presently, some special types of algebraic equations will be considered and their methods of solutions will be discussed using Vedic approach. The aim of the paper is to point out in brief the usefulness of the principles of Vedic Mathematics in solving different types of algebraic equations that may arise in the field of Science and Engineering, instead of the usual conventional methods.

I. INTRODUCTION

The most ancient ‘Religious’ scriptures of the whole world are the Vedas. ‘Veda’ is a Sanskrit word derived from the root Vid, which means to know without limit. Our ancient Vedas must be all-round, complete and perfect, throwing light on all matters for all aspiring seekers of knowledge.

Vedas are known to be four in number viz. Rigveda, Samaveda, Yajurveda and Atharvaveda. Vedas also include six Vedangas (comprising of grammar, prosody, astronomy, lexicography, etc.) and four Upavedas such as (i) Ayurveda (dealing with anatomy, physiology, hygiene, sanitary science, medical science, surgery, etc.) (ii) Dhanurveda (relating to archery and other military sciences) (iii) Gandharvaveda (the science and art of music) (iv) Sthapatyaveda (which includes engineering, architecture, etc., and also all branches of Mathematics in general).
Due to numerous invasions of India, in due course of time much of the Vedic tradition fell into disuse but in the 19th century the scholars took renewed interest in the Vedas. They however could not make sense of the mathematical portion of the Vedas contained in the *Parisista* (the appendix portion) of the *Atharvaveda*. Thereafter, a versatile scholar, Swami Bharati Krishna Tirthaji Maharaja (1884 – 1960) [1] of Govardhana Math, Puri, after long and careful investigation gave a brilliant reconstruction of the ancient mathematical system based on sixteen *Vedic Sutras* (or, mathematical aphorisms) together with a number of *Sub-sutras* (corollaries) found in the Appendix portion of the *Atharvaveda*. His wonderful research achievements in the field of Vedic Mathematics included sixteen volumes on the subject which were subsequently lost. After the confirmation of the loss, he again decided to rewrite all the sixteen volumes but could write only the introductory volume named ‘Vedic Mathematics’. His ill-health and eventual death prevented him to complete the further volumes. His introduction tells us how the mathematical reconstruction in the Veda was done and it is very astonishing as well as gratifying to find that exceedingly tough mathematical problems (with tedious and cumbersome steps of working) can be very easily solved with the simple *Vedic Sutras* and *Sub-sutras* contained in the *Parisista* of the *Atharvaveda*.

A few teachers and scholars have revived interest in the Vedic Mathematics developed by Swamiji. Dr. Narinder Puri of Roorke University studying Vedic Mathematics during 1986 – 89 stated that Vedic Mathematics with its special features has the inbuilt potential to solve the psychological problem of Mathematics i.e. anxiety. According to him, the element of choice and flexibility at each stage of problem-solving keeps the mind lively and alert to develop clarity of thought and intuition, and thereby a holistic development of the human brain automatically takes place. Dr. Michael Weinless, Chairman of the Department of Mathematics at the M.I.U, Iowa says thus: ‘Vedic Mathematics is easier to learn, faster to use and less prone to error than conventional methods. Furthermore, the techniques of Vedic Mathematics not only enable the students to solve specific mathematical problems; they also develop creativity and logical thinking’. Some great mathematicians and historians of Mathematics, Prof. G.P. Halstead, Prof. De Moregam, Prof. B.B. Dutta ([2], [3]) have expressed their whole-hearted appreciation on ancient India’s grand and glorious contributions to the progress of mathematical knowledge everywhere. However, some people are so deeply rooted in the conventional methods that they probably subconsciously reject to see the logic in unconventional Vedic methods. A.P. Nicholas, et. al.[4] have shown in their work how Vedic Mathematics offers a fresh and highly efficient approach to mathematics covering a wide range of problems starting with elementary multiplication and concluding with a relatively advanced topic such as the solution of non-linear partial differential equations. It is pointed out that Vedic scheme is not simply a collection of rapid methods but it is a unified approach, which can be swiftly learnt. Their work in the form of a book is actually a tribute to Sri Bharati Krishna Tirthaji Maharaja on the occasion of his birth centenary celebration.
II. THE VEDIC MATHEMATICAL APHORISMS OR SUTRAS

Vedic Mathematics is actually a mathematical elaboration of sixteen Sutras or sixteen mathematical formulae and thirteen Sub-sutras from the Vedas deciphered by His Holiness Jagadguru Shankaracharya Sri Bharati Krishna Tirthaji Maharaja.

The salient features of Vedic Mathematics Sutras are as follows:

- The Vedic Sutras apply to and cover each and every branch of Mathematics such as arithmetic, algebra, geometry (plane and solid), trigonometry (plane and spherical), astronomy, calculus, etc. In fact there is no part of Mathematics (pure or applied), which is beyond their jurisdiction.
- The application of the Sutras is perfectly logical and rational.
- The Sutras are easy to understand, easy to remember and easy to apply i.e., in one word the whole work is ‘mental’.
- Regarding complex problems involving a good number of mathematical computations the time taken by the Vedic method will be one-third, one-fourth, one-tenth or even a much smaller fraction of the time required by the conventional methods.
- The sums requiring numerous and cumbrous steps of working according to current methods could be answered in a single and simple step by the Vedic method.
- The computations made on the computers these days follows, in a way, the principles underlying the Sutras. Vedic Mathematics extensively exploits the properties of numbers in very practical applications, particularly in the field of computation. It gives a whole range of methods ideally suited to these properties.
- In the Vedic system, for any problem, there is always one general technique applicable to all cases; however a number of special pattern problems with some typical characteristics render them more easily solvable by means of some special Sutras and Sub-sutras, applicable to those particular types only.
- The large flexibility of the methods is reflected in the mind when approaching problems from the Vedic viewpoint.
- In Vedic scheme polynomial acts as a generalization of positional notation. The Vedic plan allows use of digits exceeding the base, in any given position, which is most unlikely in positional notation.

2.1 Vedic Solution of Algebraic Equations: In solving algebraic equations, we usually do the tedious work of practically proving a formula in question instead of taking it for granted and applying it. The Vedic method gives us the formulae (sutras) and sub-formulae (sub-sutras) which enables us to perform the necessary operation by mere application thereof. The underlying principle behind all of them is the sutra Paravartya Yojayet meaning “Transpose and adjust”. The applications of this are numerous and very useful. We shall now see how some of the wonderful Vedic sutras are used in solving different types of simple algebraic equations.

2.2 Paravartya Yojayet Sutra: This rule relating to transposition enjoins change of sign with every change of side, i.e., + becomes − and conversely; and X becomes ÷
and conversely. Further it can be extended to the transposition of terms from left to right and conversely; and from numerator to denominator and conversely, in the concerned problems.

**Type (i)**: \(3x - 4 = 2x + 6\).

\[ x = \frac{6 - (-4)}{3 - 2} = \frac{10}{1} = 10. \]

We see that the problem is of the type \(ax + b = cx + d\) from which we get by ‘transpose’ \((d - b), (a - c)\) such that \(x = \frac{d - b}{a - c}\). Here in this example \(a = 3, b = -4, c = 2, d = 6\).

**Type (ii)**: \((x + 7)(x - 6) = (x + 3)(x - 4)\).

\[ x = \frac{(3)(-4) - (7)(-6)}{7 + (-6) - 3 - (-4)} = \frac{30}{2} = 15. \]

The problem is of the type \((x + a)(x + b) = (x + c)(x + d)\), and by Paravartya Yojayet sutra we get \(x = \frac{cd - ab}{a + b - c - d}\), which is actually a trivial form of the following steps:

\[
(x + a)(x + b) = (x + c)(x + d) \\
x^2 + bx + ax + ab = x^2 + dx + cx + cd \\
x^2 + ax - dx - cx = cd - ab \\
x(a + b - c - d) = cd - ab \\
x = \frac{cd - ab}{a + b - c - d}.
\]

It is to be noted that if \(cd - ab = 0\) i.e., \(cd = ab\), the numerator becomes 0, giving \(x = 0\) as in the problem \((x + 6)(x + 3) = (x - 2)(x - 9)\), whose solution is \(x = 0\).

**Type (iii)**: \(\frac{3x + 1}{4x + 3} = \frac{13}{19}\)

\[ x = \frac{13(3) - 19(1)}{19(3) - 13(4)} = \frac{39 - 19}{57 - 52} = \frac{20}{5} = 4. \]

Here the problem is of the type \(\frac{ax + b}{cx + d} = \frac{m}{n}\) which by cross-multiplication and several steps of simplification by conventional method yields \(x = \frac{md - nb}{na - mc}\).

**Type (iv)**: \(\frac{5}{x+4} + \frac{6}{x-6} = 0\).

\[ x = \frac{-5(-6) - (-6)(4)}{5 + 6} = \frac{6}{11}, \text{by Paravartya Yojayet sutra}. \]

The problem here is of the type \(\frac{m}{x+a} + \frac{n}{x+b} = 0\) which by taking L.C.M. and proceeding by usual method yields \(x = \frac{-mb - na}{(m+n)}\).

**Other types:**
- It can be shown that for the type of equation \(\frac{m}{x+a} + \frac{n}{x+b} + \frac{p}{x+c} = 0\), the solution is \(x = \frac{-mb - na - pab}{m(b + c) + n(c + a) + p(a + b)}\), if \(m + n + p = 0\).
For the equation \( \frac{m}{x+a} + \frac{n}{x+b} = \frac{m+n}{x+c} \), it can be found after some steps of conventional working that 
\[ x = \frac{mb(a-c)+na(b-c)}{m(c-a)+n(c-b)} . \]
By Paravartya rule we can easily remember the formula.

2.3 Samuccaye Sutra: The term ‘Samuccaya’ has several meanings under different contexts. The sutra ‘Sunyam Samyasamuccaye’ says that the ‘Samuccaya is zero’ i.e., it should be equated to zero.

Sun yam Samyasamuccaye sutra in linear and quadratic equations.

Let us now see the different interpretations of ‘Samuccaya’ in some linear and quadratic equations as follows:

(i) ‘Samuccaya’ is interpreted as a term which occurs as a common factor in all the terms concerned:

We consider here some simple examples to understand the sutra.

Consider \( 3x + 7x = 4x + 5x \)

Here the equation has \( x \) in all its terms and hence by the sutra it is zero, i.e.,
\[ x = 0. \]

Otherwise conventionally, we have to work as
\[
3x + 7x = 4x + 5x \\
10x = 9x \\
10x − 9x = 0 \\
x = 0
\]

\[ 6(x + 1) = 2(x + 1). \]

Now here the Samuccaya is \( x + 1 \) and hence by the sutra \( x + 1 = 0 \) which gives \( x = −1. \)

(ii) Now we interpret ‘Samuccaya’ as product of independent terms in expressions like \( (x + a)(x + b) \):

\[ (x + 3)(x + 4) = (x − 2)(x − 6) \]

Here Samuccaya is \( 3 \times 4 = 12 = −2 \times −6 \). Since it is same, by the sutra we can say \( x = 0. \)

We have seen before that this type of equation can also be solved using Paravartya method.

(iii) We now interpret ‘Samuccaya’ as the sum of the denominators of two fractions having the same numerical numerator.

\[ \frac{1}{3x−2} + \frac{1}{2x−1} = 0. \]

Here we can directly put the Samuccaya i.e., sum of the denominators \( = 0, \)
\[ i.e., 3x − 2 + 2x − 1 = 5x − 3 = 0 \] giving \( 5x = 3 \) i.e., \( x = 3/5 \) is the solution.

Whereas by our current method we get the solution by taking L.C.M. and proceeding as follows:

\[
\frac{(2x − 1) + (3x − 2)}{(3x − 2)(2x − 1)} = 0
\]
\[
\frac{5x - 3}{(3x - 2)(2x - 1)} = 0
\]
\[
\frac{5x - 3}{3x - 2} = \frac{5x - 3}{2x - 1} = 0
\]
\[
5x - 3 = 0
\]
\[
5x = 3
\]
\[
x = \frac{3}{5}
\]

The *sutra* is applicable for all problems of the type

\[
\frac{m}{ax+b} + \frac{m}{cx+d} = 0, \quad s.t. \quad \text{the Samuccaya is } ax + b + cx + d \quad \text{and solution is thus}
\]

\[
x = \frac{-(b+d)}{(a+c)}.
\]

(iv) Here ‘*Samuccaya*’ is interpreted as a *combination or total*:

If the sum of the numerators and the sum of the denominators are equal, then the solutions will be obtained by

\[
N_1 + N_2 = D_1 + D_2 \quad \text{(sum of the numerators)} = 0.
\]

In case of problems leading to quadratic equations, \(N_1 \sim D_1 = N_2 \sim D_2 = 0\) along with the above give the two solutions.

\(\Rightarrow\) Consider

\[
\frac{5x + 7}{5x + 12} = \frac{5x + 12}{5x + 7}
\]

Here \(N_1 + N_2 = D_1 + D_2 = 10x + 19\).

Hence from *Sunya Samuccaya* we get \(10x + 19 = 0\) s.t. \(x = -19/10\).

It is to be noted here that if \(N_1 + N_2 = k(D_1 + D_2)\), where \(k\) is a constant then also by removing the numerical constant, we can proceed as usual.

\(\Rightarrow\)

\[
\frac{2x + 3}{4x + 5} = \frac{x + 1}{2x + 3}
\]

Here \(N_1 + N_2 = 3x + 4\) and \(D_1 + D_2 = 6x + 8 = 2(3x + 4) = 2(N_1 + N_2)\)

So removing the numerical factor 2 we take \((3x + 4) = 0\). s.t. \(x = -4/3\).

\(\Rightarrow\)

\[
\frac{3x + 4}{6x + 7} = \frac{5x + 6}{2x + 3}
\]

By our current method it can be solved as follows:

\((3x + 4)(2x + 3) = (5x + 6)(6x + 7)\)

Or, \(6x^2 + 17x + 12 = 30x^2 + 35x + 36x + 42\)

Or, \(24x^2 + 54x + 30 = 0\)

Or, \(4x^2 + 9x + 5 = 0\)

Or, \(4x^2 + 4x + 5x + 5 = 0\)

Or, \(4x(x + 1) + 5(x + 1) = 0\)

Or, \((4x + 5)(x + 1) = 0\)

Or, \(x + 1 = 0, \ 4x + 5 = 0\)

\(i.e., x = -1, x = -5/4\).

 Whereas by applying *Samuccaya Sutra* we have in this case

\(N_1 + N_2 = D_1 + D_2 = 8x + 10 = 0\) \(or, \quad x = -5/4\)

and \(N_1 \sim D_1 = N_2 \sim D_2 = 3x + 3 = 0\) \(or, \quad x = -1\).

(v) ’*Samuccaya*’ with the same sense but with a different context and application.
Suppose we have to solve the another type of equation such as \( \frac{1}{x-7} + \frac{1}{x-9} = \frac{1}{x-6} + \frac{1}{x-10} \).

Conventionally we do this by transposing two of the terms so that each side has a plus term and a minus term, take the L.C.M. of the denominators, cross-multiply, equate the denominators, expand them, transpose and so on and so forth. After almost 10 steps of working we get 8 as an answer.

However, the Samuccaya sutra tells us that if the other elements are equal and if the sum-total of the denominators on the L.H.S. and their total on the R.H.S. be the same, then the solution will be obtained by equating the total to zero.

Now in this case \( D_1 + D_2 = 2x - 16 \), and \( D_3 + D_4 = 2x - 16 \).

\[ \therefore 2x - 16 = 0 \]  \[ \text{i.e.,} \quad x = 8. \]

\[ \Rightarrow \frac{1}{x-b} - \frac{1}{x-b-d} = \frac{1}{x-c+d} - \frac{1}{x-c} \]

The given equation can be rewritten by transposition as

\[ \frac{1}{x-b} + \frac{1}{x-c+d} = \frac{1}{x-c} + \frac{1}{x-b-d}. \]

\[ D_1 + D_2 = D_3 + D_4 = 2x - (b + c) \]

\[ \therefore \text{the solution is given by} \quad D_1 + D_2 = 0 \Rightarrow 2x - (b + c) = 0 \]

\[ \text{i.e.,} \quad x = \frac{1}{2}(b + c). \]

If this equation with so many literal coefficients involved is solved with the current method, the labour entailed over the L.C.M.’s, the multiplications, etc., would have been terrific and the time taken would have been proportionate too. By this Vedic method, the equation is solved at sight.

\[ \Rightarrow \frac{x-2}{x-3} \div \frac{x-3}{x-4} = \frac{x-2}{x-3} + \frac{x-3}{x-4} = \frac{x-4}{x-5} + \frac{x-3}{x-4} \]

On dividing the numerators by the denominators, we have

\[ 1 + \frac{1}{x-3} + 1 + \frac{1}{x-4} = 1 + \frac{1}{x-2} + 1 + \frac{1}{x-5} \]

or \[ \frac{1}{x-3} + \frac{1}{x-4} = \frac{1}{x-2} + \frac{1}{x-5} \]

Since all the numerators are equal and \( D_1 + D_2 = D_3 + D_4 = 2x - 7 \), by the sutra the solution is given by \( 2x - 7 = 0 \) i.e., \( x = \frac{7}{2} \).

The above process of division can be mentally performed very easily as follows:

1st step: \( \frac{x}{x} + \frac{x}{x} = \frac{x}{x} + \frac{x}{x} (1 + 1 = 1 + 1) \)

2nd step: Applying Paravatya method mentally the numerators of the remainder will be 1 in each of the four cases.

So with the help of these tests we know that the other elements are the same and as \( D_1 + D_2 = D_3 + D_4 \), so the solution is \( 2x - 7 = 0 \) or, \( x = \frac{7}{2} \).

- **Sunyam Samyasamuccaye sutra in cubic equations.**

\[ \Rightarrow \text{Consider the problem} \quad (x - 4)^3 + (x - 6)^3 = 2(x - 5)^3. \]
For the solution by the traditional method we follow the steps as given below:

\[(x - 4)^3 + (x - 6)^3 = 2(x - 5)^3\]
\[x^3 - 12x^2 + 48x - 64 + x^3 - 18x^2 + 108x - 216 = 2(x^3 - 15x^2 + 75x - 125)\]
i.e., \[2x^3 - 30x^2 + 156x - 280 = 2x^3 - 30x^2 + 150x - 250\]
i.e., \[156x - 280 = 150x - 250\]
i.e., \[6x = 30\]
i.e., \[x = 30 / 6 = 5\]

Now if we observe the problem in the Vedic sense, we have
\[(x - 4) + (x - 6) = 2x - 10.\]
Taking out the numerical factor 2 we have \((x - 5)\), which is the factor under the cube on R.H.S.

In such a case “Sunyam samya Samuccaye” formula gives that \(x - 5 = 0\).

Hence \(x = 5\).

⇒ Let us now think of solving the problem

\[(x-249)^3 + (x+247)^3 = 2(x-1)^3.\]

The traditional method will be horrible even to think of!!

But \((x - 249) + (x + 247) = 2x - 2 = 2(x - 1)\) and \((x - 1)\) is a factor on R.H.S. of the equation. So, we state that \(x - 1 = 0\) by the ‘sutra’.

\[\therefore x = 1\] is the solution. Thus no cubing or any other mathematical operations are required.

Algebraic Proof of the Sutra:
Consider \((x - 2a)^3 + (x - 2b)^3 = 2(x - a - b)^3\)

It is clear that \(x - 2a + x - 2b = 2x - 2a - 2b = 2(x - a - b)\)

Now the given equation becomes
\[x^3 - 6x^2a + 12xa^2 - 8a^3 + x^3 - 6x^2b + 12xb^2 - 8b^3 = 2(x^3 - 3x^2a - 3x^2b + 3xa^2 + 3xb^2 + 6axb - a^3 - 3a^2b - 3ab^2 - b^3)\]
\[= 2x^3 - 6x^2a - 6x^2b + 6xa^2 + 6xb^2 + 12xab - 2a^3 - 6a^2b - 6ab^2 - 2b^3\]

Cancelling the common terms on both sides, we get
\[12xa^2 + 12xb^2 - 8a^3 - 8b^3 = 6xa^2 + 6xb^2 + 12xab - 2a^3 - 6a^2b - 6ab^2 - 2b^3\]
i.e., \[6xa^2 + 6xb^2 - 12xab = 6a^3 + 6b^3 - 6a^2b - 6ab^2\]
i.e., \[6x(a^2 + b^2 - 2ab) = 6[a^3 + b^3 - ab(a + b)]\]
i.e., \[x(a - b)^2 = [(a + b)(a^2 + b^2 - ab) - (a + b)ab] = (a + b)(a^2 + b^2 - 2ab) = (a + b)(a - b)^2\]
\[\therefore x = a + b.\]
\[
\frac{(x + 2)^3}{(x + 3)^3} = \frac{x + 1}{x + 4}
\]

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With the conventional procedures we proceed as follows:

\[
\frac{x^3 + 6x^2 + 12x + 8}{x^3 + 9x^2 + 27x + 27} = \frac{x + 1}{x + 4}
\]

Now by cross multiplication,

\[
(x + 4)(x^3 + 6x^2 + 12x + 8) = (x + 1)(x^3 + 9x^2 + 27x + 27)
\]

\[x^4 + 6x^3 + 12x^2 + 8x + 4x^3 + 24x^2 + 48x + 32 = x^4 + 9x^3 + 27x^2 + 27x + x^3 + 9x^2 + 27x + 27\]

\[x^4 + 10x^3 + 36x^2 + 56x + 32 = x^4 + 10x^3 + 36x^2 + 54x + 27\]

\[56x + 32 = 54x + 27\]

\[56x - 54x = 27 - 32\]

\[2x = -5\]

\[\therefore x = -5/2\]

Now under Vedic consideration, we observe that \((N1 + D1)\) within the cubes on L.H.S. is \(x + 2 + x + 3 = 2x + 5\) and \((N2 + D2)\) on the right hand side is \(x + 1 + x + 4 = 2x + 5\), s.t. by the *sutra* we have \(2x + 5 = 0\), *i.e.*, \(x = -5/2\).

### 2.4 Sunyam Anyat Sutra:

Suppose we have to solve the equation

\[
\frac{2}{x+2} + \frac{3}{x+3} = \frac{4}{x+4} + \frac{1}{x+1}.
\]

The nature of the characteristics of this special type will be recognizable with the help of the usual old test and additional new test which are as follows:

Here we see that \(\frac{2}{1} + \frac{3}{1} = \frac{4}{1} + \frac{1}{1}\) and \(\frac{2}{2} + \frac{3}{3} = \frac{4}{2} + \frac{1}{1}\).

In all such cases *Sunyam Anyat Sutra* says that one root is 0 and Sunyam Samuccaye *sutra* says

\(D1 + D2 = D3 + D4 = 0\), *i.e.*, \(2x + 5 = 0\) or, \(x = -5/2\).

Thus, the required solutions are \(0, -5/2\).

### 2.5 Puranapuranabhyaam Sutra:

The *Sutra* can be taken as Purana - Apuranabhyaam which means by the completion or non - completion. Purana is well known in the present system. We can see its application in solving the roots for general form of quadratic equation as follows:

Consider the quadratic equation \(ax^2 + bx + c = 0\)

\[x^2 + \frac{(b/a)x + c/a}{a} = 0 \text{ (dividing by } a)\]

\[x^2 + (b/a)x = -c/a\]

Completing the square (i.e., purana) on the L.H.S., we have

\[x^2 + (b/a)x + \left(\frac{b^2}{4a^2}\right) = -c/a + \left(\frac{b^2}{4a^2}\right)\]

\[\left[x + \left(\frac{b}{2a}\right)\right]^2 = \frac{(b^2 - 4ac)}{4a^2}\]

Proceeding in this way we finally get \(x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}\).

Now we apply *purana* to solve the following problems:

*Example 1*: \(x^2 + 6x^2 + 11x + 6 = 0\).

*Since* \((x + 2)^3 = x^3 + 6x^2 + 12x + 8\), 

*Adding* \((x + 2)\) to both sides, we get...
\[ x^3 + 6x^2 + 11x + 6 + x + 2 = x + 2 \]
i.e., \[ x^3 + 6x^2 + 12x + 8 = x + 2 \]
i.e., \((x + 2)^3 = (x + 2)\)
this is of the form \(y^3 = y\) for \(y = x + 2\),
whose solution is \(y = 0, y = 1, y = -1\)
i.e., \(x + 2 = 0, 1, -1\)
which gives \(x = -2, -1, -3\).

\[ \Rightarrow \text{Example 2: } x^3 + 8x^2 + 17x + 10 = 0 \]
We know \((x + 3)^3 = x^3 + 9x^2 + 27x + 27\)
so adding on the both sides, the term \((x^2 + 10x + 17)\), we get
\[ x^3 + 8x^2 + 17x + x^2 + 10x + 17 = x^2 + 10x + 17 \]
i.e., \(x^3 + 9x^2 + 27x + 27 = x^2 + 6x + 9 + 4x + 8 \)
i.e., \((x + 3)^3 = (x + 3)^2 + 4(x + 3) - 4 \)
i.e., \(y^3 = y^2 + 4y - 4\) for \(y = x + 3\)
i.e., \(y = 1, 2, -2\).
Hence \(x = -2, -1, -5\).

Thus \textit{purana} is helpful in factorization.

Further, \textit{purana} can be applied in solving Biquadratic equations also.

\textbf{2.6 Antyayoreva sutra:} 'Atyayoreva' means 'only the last terms'. This is useful in solving simple equations of the type that follows. The type of equations are those whose numerator and denominator on the L.H.S. bearing the independent terms stand in the same ratio to each other as the entire numerator and the entire denominator of the R.H.S. stand to each other.

Let us have a look at the following example:

\[ \frac{x^2 + 2x + 7}{x^2 + 3x + 5} = \frac{x + 2}{x + 3} \]

In the conventional method we proceed as follows:

\[ \frac{x^2 + 2x + 7}{x^2 + 3x + 5} = \frac{x + 2}{x + 3} \]
\( (x + 3)(x^2 + 2x + 7) = (x + 2)(x^2 + 3x + 5) \)
\[ x^3 + 2x^2 + 7x + 3x^2 + 6x + 21 = x^3 + 3x^2 + 5x + 2x^2 + 6x + 10 \]
\[ x^3 + 5x^2 + 13x + 21 = x^3 + 5x^2 + 11x + 10 \]

Canceling like terms on both sides

\[ 13x + 21 = 11x + 10 \]
\[ 13x - 11x = 10 - 21 \]
\[ 2x = -11 \]
\[ x = -11 / 2 \]

Now we solve the problem using \textit{anatyayoreva}.

\[ \frac{x^2 + 2x + 7}{x^2 + 3x + 5} = \frac{x + 2}{x + 3} \]

We observe that
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\[ x^2 + 2x = x(x + 2) \]
\[ x^2 + 3x = x(x + 3) \]

This is according to the condition in the *sutra*. Hence from the *sutra*

\[ x + 2 = 7 \]
\[ x + 3 = 5 \]
\[ 5x + 10 = 7x + 21 \]
\[ 7x - 5x = -21 + 10 \]
\[ 2x = -11 \]
\[ x = -\frac{11}{2} \]

Its algebraic proof can be easily given.

It is to be noted here that in general the *Urdhvatiryak sutra* in Vedic Mathematics extends naturally to the solution of cubic equations and to higher order equations. However, much work remains to be done, especially regarding its convergence.

2.7 Vilokanam sutra: The Sutra 'Vilokanam' means 'Observation'. Generally we come across problems which can be solved by mere observation. But we follow the same conventional procedure and obtain the solution. But the hint behind the Sutra enables us to observe the problem completely and find the pattern and finally solve the problem by just observation.

⇒ Let us take the equation \( x + \left( \frac{1}{x} \right) = \frac{5}{2} \)

Without noticing the logic in the problem, the conventional process tends us to solve the problem in the following way:

\[
\begin{align*}
    x + \frac{1}{x} &= \frac{5}{2} \\
    x^2 + 1 &= \frac{5}{2} \\
    2x^2 + 2 &= 5x \\
    2x^2 - 5x + 2 &= 0 \\
    2x^2 - 4x - x + 2 &= 0 \\
    2(x - 2) - (x - 2) &= 0 \\
    (x - 2)(2x - 1) &= 0 \\
    x - 2 &= 0 \text{ gives } x = 2 \\
    2x - 1 &= 0 \text{ gives } x = \frac{1}{2}
\end{align*}
\]

But by *Vilokanam* i.e., observation \( x + \frac{1}{x} = \frac{5}{2} \) can be viewed as

\[ x + \frac{1}{x} = 2 + \frac{1}{2} \text{ giving } x = 2 \text{ or } \frac{1}{2}. \]

⇒ \[ \frac{x}{x+2} + \frac{x+2}{x} = \frac{34}{15} \]

In the conventional process, we have to take L.C.M, cross-multiplication, simplification and factorization.

But according to *Vilokanam*, we have

\[ \frac{34}{15} = \frac{9+25}{5 \times 3} = \frac{3}{5} + \frac{5}{3} \]
III. CONCLUSION

In this article it has been precisely shown that how very economically and with least effort the wonderful Vedic sutras can be used to solve variety of algebraic equations that arise in the field of science. An attempt has been made only to make one familiar with a few special methods for solving different types of algebraic equations. One can compare and contrast both the Vedic and conventional methods and clearly realize the beauty, simplicity and resourcefulness in Vedic Mathematical systems. Although Swami Bharati Krishna Tirthaji has open the doors and thrown light to the vastness and versatility of Vedic mathematics, much research work is still required to be done regarding the Vedic sutras and their applications, especially in the world of equations. However, serious and sincere work by scholars and research workers is still continuing in this field both in our country and abroad. Sri Sathya Sai Veda Pratishthan intends to bring about more volumes covering the aspects now left over, and also elaborating the contents of ‘Vedic Mathematics’ by Swami Bharati Krishna Tirthaji Maharaja. Modern day computations will become easier to handle following the concepts of Vedic Mathematics. May India continue to uphold her ancient mathematical tradition and prosper.

IV. REFERENCES


TO CITE THIS PAPER