New Single Asymmetric Error Correcting Codes with a Partitioning of Binary Vectors

V.K. Sharma, R.K. Dahiya
Department of mathematics
1 S.G.N. Khalsa PG College, Sri Ganganagar (Rajasthan)
2 Govt. PG College for women, Bhodia Khera (Fatehabad)

Abstract
In this research paper new single Error Correcting codes with a new partitioning of binary vectors are presented. These codes are better than existing codes [1, 2, 3, 4, 5] when code length n is 22 or above except at n=9. But we have proved this contention for length up to 22. A new partitioning method of binary vectors is also presented, which is the same for all the values of n and a more useful new partitioning has also been obtained.

Keywords: Error Correcting Codes, Binary Vectors, New Partitioning, Hamming Distance, Arbitrariness

1 Introduction
New single asymmetric error–Correcting codes of asymmetric distance 2 are presented. The asymmetric distance between two binary vectors $x$ and $y$, of length $n$, as given by Al-Bassam, Venkatesan and Al-Muhammad (1997):
$$\Delta(x, y) = \max\{ N(x, y), N(y, x) \}$$
Where $N(x, y) = | \{ i: x_i = 1 \text{ and } y_i = 0 \} |$ and the minimum asymmetric distance of a code $C$ is defined by
$$\Delta(C) = \min\{ \Delta(x, y): x, y \in C; x \neq y \},$$
The function $\Delta$ is used to measure both the asymmetric distance between two binary vectors and to measure the asymmetric distance between a set of vectors (or a code). One may use the notation $\Delta(\{x, y\})$ instead of $\Delta(x, y)$ for consistency; however, for simplicity the latter form is used here.

The Hamming distance between two vectors can be defined as
$$D(x, y) = | \{ i: x_i \neq y_i \} |.$$
In this Chapter, we have constructed the codes by Cartesian product of two smaller codes as
$$C(codes) = A_1 \times B_1 \cup A_2 \times B_2 \cup A_3 \times B_3 \ldots \ldots$$

Table 1: New Single Asymmetric Error-Correcting Codes

<table>
<thead>
<tr>
<th>$n$</th>
<th>proposed Codes</th>
<th>existing Codes</th>
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<tr>
<td>3</td>
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<td>$2^a$</td>
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<tr>
<td>4</td>
<td>4</td>
<td>$4^a$</td>
</tr>
</tbody>
</table>

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For an easy reference, Table 1, lists the new codes and the best known codes given by Varshamov (1973), Kim and Freiman (1959), Delsarte and Piere (1981), Zhang and Xia (1992), Bassam, Venkatesan and Muhammad (1997).

2 Construction Method

For the construction of codes, we give the following properties about the partitions of A and B of C.

Property I: Let A be the set of all the $2^p$ binary vectors of length p such as:

(i) $A = A_1 \cup A_2 \cup A_3 \cup ... \cup A_p$ where $A_1, A_2, A_3, ... A_p$ are the partition of A.

(ii) $A_i \cap A_j = \emptyset$ and $A_i = A$, such that $|A_i| \geq 2$ for $1 \leq i \leq p$.

Property II: Let B be the set of all the $2^{q-1}$ binary vectors of length q such as:

(i) $B = B_1 \cup B_2 \cup B_3 \cup ... \cup B_q$ where $B_1, B_2, B_3, ... B_q$ are the partition of B.

(ii) $B_i \cap B_j = \emptyset$ and $B_i = B$, such that $|B_i| \geq 2$ for $1 \leq i \leq q$.

Let $C$ be the codes obtained by the Cartesian product of $A_i \times B_i$ i.e.

$C(\text{codes}) = A_1 \times B_1 \cup A_2 \times B_2 \cup A_3 \times B_3 \cup ... \cup A_p \times B_q$

Where $p \neq q$, some $A_i$’s or $B_i$’s will be empty; the code $C$ has

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<td>652^i</td>
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<td>28032^o</td>
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<td>53856^p</td>
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<td>101576^q</td>
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<td>22</td>
<td>203567*</td>
<td>195700^r</td>
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</table>

*a* Code by Varshamov (1973)

*b* Code by Kim and Freiman (1959)

*c* Code by Delsarte and Piere (1981)

*d* Code by Zhang and Xia (1992)

*e* Code by Bassam, Venkatesan and Muhammad (1997)

* Proposed code improving the existing code.
Theorem 1: The code \( C \), obtained in (1) of length \( n = p + q \) is a single asymmetric error-correcting code.

Theorem 2: Let \( q = r \times 2^i \), where \( 1 \leq r \leq 6 \) and \( i \geq 1 \). One can construct a code \( C \) of length \( = 2q - 2 \), containing at least \( \left\lceil \frac{2^n}{n} \right\rceil \) codewords (Bassam, Venkatesan and Muhammandi (1997)).

Example 1: To construct a single asymmetric error correcting code with \( n = 7 \), let \( p = 3 \), \( q = 4 \), then

\[ A = \{000,001,010,011,100,101,110,111\} \text{ can be partitioned into } A_1 = \{000,001,011\} , \quad A_2 = \{010,101\} , \quad A_3 = \{100,110\} , \quad A_4 = \{111\} \text{ and } \quad B = \{0000,0011,0101,0110,1001,1010,1100,1111\} \text{ can be partitioned into } \]

\[ B_1 = \{0000,0001,1100,1111\} , \quad B_2 = \{0101,1010\} , \quad B_3 = \{0110,1001\} . \text{ Now we construct a code } \]

\[ C = A_1 \times B_1 \cup A_2 \times B_2 \cup A_3 \times B_3 \]

\[ = 3 \times 4 \cup 2 \times 2 \cup 2 \times 2 \]

\[ = 20 \text{ codewords} \]

<table>
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<th>( A_1 \times B_1 = 3 \times 4 = 12 )</th>
<th>( A_2 \times B_2 = 2 \times 2 = 4 )</th>
<th>( A_3 \times B_3 = 2 \times 2 = 4 )</th>
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<td>001 1100</td>
<td>100 0101</td>
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<td>000 1100</td>
<td>011 0000</td>
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<tr>
<td>001 0011</td>
<td>011 1111</td>
<td>110 1010</td>
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</tbody>
</table>

3 Partitioning

To maximize the size of the code \( C \) of length \( n \), we must choose appropriate values of \( p \) and \( q \), so that \( n = p + q \).

The code words derived from \( p \) are partitioned in classes where \( A_1, A_2, A_3, ... A_p \), and \( q \) in \( B_1, B_2, B_3, ..., B_q \) classes. We multiply the higher class of \( A_i \) with higher class of \( B_j \) and lower of \( A_i \) to lower of \( B_j \) and so on.

For all \( i = 1,2,3, ... p' \) and \( j = 1,2,3, ... q' \). All the

Must satisfy \( |A_j| \geq |A_{j+1}| \) for all \( j \).

Now we partition \( p \) or \( q \), with the range of 1 to 14 in different classes according to weight as:
Table II Partitioning of vectors according to their weight distribution in different classes

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<th>w→ P or q↓</th>
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<th>3</th>
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<th>5</th>
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</table>

A’s Partitions:

According to property I, A will contain $2^p$ binary vectors of length $p'$. Partitioning of these vectors according to their weight distribution in different classes is shown in table II. Al-Bassam, Venkatesan and Al-Muhammad (1997), partitioned $2^p$ binary vector into $p + 1$ classes.
Example II: Let \( p = 5 \). There will be \( p + 1 \) classes, i.e. 6 and the total number of binary vector \( 2^5 = 32 \). In the case of weight 2 class, 10 is equally distributed in 6 classes and the remaining 4 in first four. In the case of weight 3 class, 10 is equally distributed over five classes and so on. The distribution of binary vectors is shown in table IIA.

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</table>

The above mention technique is applied to all the partitioning of \( p \) for different length. Whereas the partitioning technique of Al-Bassam, Venkatesan and Al-Muhammadi (1997), suffers from arbitrariness, the authors thus have used a uniform technique for all the partitions.

<table>
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B’s Partitions:
The property II defines that set B will have all the even weight vectors of length $q$. The division of all even weight vectors of length $q$ in different category is shown in table IV.

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When $q$ is even, the number of classes will be $q - 1$ and when $q$ is odd the number of classes will be $q$.

Example III: Let $q = 7$. The total number of classes is 7 and the total number of even weight vector is $2^{7-1} = 2^6 = 64$. The author have applied the same technique to obtain the partition of B. in the...
case of weight 4 class, 35 is equally distributed in 7 classes. In the case of weight 2 class, 21 are equally distributed over six classes and remaining 3 in the first three. The distribution of binary vectors will be shown in Table IIIA.

<table>
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<tr>
<th></th>
<th>B_1</th>
<th>B_2</th>
<th>B_3</th>
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Applying the above techniques, the author obtained all the partition of B for different value of q as shown in the table V.

Table V: All the Partition of B for Different Value of q

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5 Conclusion

The proposed new codes by taking the appropriate value of \( p \) and \( q \) improve the existing codes except when \( n = 9 \). The codes satisfy the lower bounds i.e. \( \left[ \frac{2^n}{n} \right] \) and the upper bounds. The author have presented codes only up to \( n = 22 \). The construction procedure may also be applied for \( n > 22 \).

<table>
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<th>( n )</th>
<th>( p )</th>
<th>( q )</th>
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<th>Proposed Codes</th>
<th>Lower Bounds</th>
<th>Upper Bounds</th>
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</table>

Table VI: The values of \( p \) and \( q \) that produce codes for \( n \leq 22 \)
The values of \( p \) and \( q \) that produce codes for \( n \leq 22 \) are listed in table VI. The author observes that \( q \) is an even value and \( p \) is the largest integer less, which is then, or equal to \( q \). The value of \( p \) is equal to \( q \) at five places i.e. \( n = 4, 8, 12, 16, 20 \). At \( n = 12 \), two values of \( p \) and \( q \) are taken, when \( p = q = 6 \) it gives 379 code words and when \( p = 4 \) and \( q = 8 \), it gives 342 code words. Thus both values improve the existing codes.

### References


