Reliability Evaluation of Mixed Configuration Models Using Fault-Tree Analysis Approach

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Abstract

In Reliability Evaluation studies, engineering systems may form various types of configurations. If the Reliability factor or the probability of failure of the system is to be determined, we will find that it is very difficult to analyze the system in its entirety. The failure of the system as a whole can be attributed to the failure of one or more components of the system not functioning in the stipulated manner. In practice, the system is broken down into subsystems and elements whose individual reliability factors can be estimated or determined. In this paper, we present some mixed models involving series and parallel configurations and determine the system reliability from the reliability factors of the subsystem and elements. System Reliability is evaluated using fault-tree analysis approach and comparison study has been made. Numerical illustration is provided.

Keywords: Reliability, Series and Parallel Configuration, Hazard Rate, Mean time to failure, Fault-tree Analysis Approach.

1. Introduction

Reliability as a human attribute has been praised for a very long time. For technical systems, however, the Reliability concept has not been applied for more than 60 years. It emerged with a technological meaning just after World War I and was then used in connection with comparing operational safety of one, two, and four engine airplanes [8]. A more detailed history of Reliability technology is presented, for example, by Knight (1991) and Villemeur (1988).

A system is a set of components arranged to accomplish a purpose or purposes under a given set of conditions. From the Statistical information available on the failure and repair cycles of the components, we determine the reliability characteristics of the system. Non-maintained systems are
treated extensively in the literature. Restorative action is initiated immediately after the failure and Reliability modeling and evaluation of numerical values of Reliability measures can be obtained either through simulation or by solving mathematical models. The reliability of technological devices is very important especially for Medical engineering. Toporkov [12] reported that up to 80% of medical equipments presently used in public health organizations is worn-out or obsolete, which makes it difficult to guarantee not only reliability and efficiency but also safety of medical equipment. Lucian and Leape [6] point out the evidences from various sources and indicates that a number of hospitalized patients suffer treatment- caused injuries most of which are from systems failure.

The history of the Reliability field may be traced back to the 1930s and 1940s, when the probability concepts were applied to electric power generation related problems [7,11,2,1] and Germans applied the basic Reliability concepts to improve reliability of their V1 and V2 rockets [3, 10]. Ever since those days, many new developments have taken place, and the field has branched out into many specialized areas: software reliability, human reliability, mechanical reliability, power system reliability, structural reliability, etc. Comprehensive lists of publications on almost all of the reliability areas are given in references [4] and [5]. H.L. Williams pointed out that human element reliability must be included in the system-reliability of prediction; otherwise the predicted value would not represent the real system-reliability prediction. A study conducted in 1960 revealed that human error is responsible for 20-50% of all equipment failures [9]. The first step in developing a Reliability model is to define and describe the system and its requirements. The system must be categorized into major subsystems and the function of each subsystem and interface between them defined. In terms of developing a model for the Reliability calculations, it may sometimes be preferable to group the components from one natural subsystem to another to create independent sub-systems as this facilitates reliability calculations. Reliability modeling is the process of predicting or understanding the Reliability of a component or system prior to its implementation. Two types of analysis often used to model a System Reliability behavior are Fault-tree Analysis and Reliability Block diagrams.

Over the years, many techniques and methods have been developed to prevent systems failure. In this paper, we deal with models of mixed parallel and series configurations and derive their system reliabilities, mean time to failure and hazard rates. Also, comparison study has been made to analyze their reliabilities. The rest of the paper is organized as follows. In section 2, we present the failure distribution, reliability related definitions, hazard rate and the Fault tree Analysis approach. In section 3, we analyze the General - parallel and series configurations and their reliability network. In section 4, we present two different models of mixed parallel and series systems. Finally a comparative study of the models is made and conclusion is given in section 5.

2. Reliability-Related Concepts

2.1 Failure Distribution – Exponential
This is the most widely used distribution in reliability theory and is one of the simplest distributions used in performing practically inclined reliability analysis. The probability density function is defined by

\[ f(t) = \lambda e^{-\lambda t}, t \geq 0, \lambda > 0 \]

where \( t \) is time, \( \lambda \) is the distribution parameter. In Human reliability work, it is known as the error rate. The corresponding cumulative density function is denoted by \( F(t) \).
2.2 Definitions

Reliability is defined by

\[ R(t) = 1 - F(t) \]
\[ = 1 - \int_{-\infty}^{t} f(x) \, dx \]

or \[ R(t) = \int_{t}^{\infty} f(x) \, dx, \quad f(x) = \lambda e^{-\lambda x} \]

or \[ R(t) = e^{-\int_{0}^{t} \lambda(t) \, dt} \]

where \( R(t) \) is the reliability at time \( t \)

\( \lambda(t) \) is the hazard rate or time dependent failure rate.

Mean time to failure is defined by

\[ MTTF = \int_{0}^{\infty} R(t) \, dt \]

Hazard Rate is defined by

\[ \lambda(t) = -\frac{1}{R(t)} \left( d\frac{R(t)}{dt} \right) \]

2.3 Fault tree Analysis (FTA) approach

FTA is a top-down approach of a system analysis that is used to determine the possible occurrence of undesirable events or failures. Over the years, the method has gained favor over other reliability analysis approaches because of its versatility in degree of detail of complex systems. There are many symbols used to construct fault trees. The basic four symbols are

- It denotes a fault event that occurs from the logical combination of fault events through the input of logic gates such as OR and AND.

- It denotes a basic fault event

- It denotes the output fault event if one or more of input fault events occur.
2.4 Series Configuration

It is the simplest and probably the most commonly occurring or assumed configuration in reliability evaluation of engineering systems. The success of the systems depends on the success of all its elements. If any one of the elements fails, the system fails. Fig. 1 shows the block diagram of a series system.

Series configuration

![Block diagram of K unit series system](image)

For independent unit, the reliability of the series system is given by

\[ R_s = R_1 R_2 R_3 \ldots R_k \]

where

- \( R_s \) - system reliability
- \( k \) - no. of independent units in series
- \( R_i \) - unit i reliability, \( i = 1, 2, \ldots, k \)

For constant failure rate of unit i,

\[ R_i(t) = e^{-\lambda t} \]
Reliability of the system at time \( t \)

\[
R_s(t) = e^{-\sum_{i=1}^{k} \lambda_i t}
\]

The mean time to failure of the system yields

\[
MTTF_s = \frac{1}{\lambda_s}
\]

where \( \lambda_s(t) = \sum_{i=1}^{k} \lambda_i \) is the series system hazard or failure rate.

### 2.5 Parallel Configuration

In this case, all units are active and at least one unit must perform successfully for the system success. Fig.2 shows the block diagram of a parallel system, each block represents a unit. In parallel system reliability is expressed by

\[
R_p = 1 - \left(1 - R_1 \right) \left(1 - R_2 \right) \cdots \left(1 - R_n \right)
\]

where \( R_p \) - Parallel system reliability

\( n \) - no. of independent units

\( R_i \) - is the unit i reliability, for \( i = 1, 2, 3, \ldots n \)

Parallel configuration

For constant failure rates of parallel units,

\[
R_p(t) = 1 - \left(1 - e^{-\lambda t} \right) \left(1 - e^{-\lambda t} \right) \cdots \left(1 - e^{-\lambda t} \right)
\]

where \( R_p(t) \) is the parallel system reliability at time \( t \).

\( \lambda \) is the constant failure rate.

For identical units \( \lambda = \lambda \)

\[
R_p(t) = 1 - \left(1 - e^{-\lambda t} \right)^n
\]

### 3. Mixed Configurations

#### 3.1 General Series -Parallel Configuration

The System consists of stage 1, stage 2... stage k connected in series. Each stage contains a number of redundant elements, stage 1 consisting of \( n_1 \) redundant elements connected in parallel. The reliability of the system is the product of the reliabilities of each stage. Stage i with \( n_i \) elements will have the reliability
3.2 General Parallel-Series Configuration

The system consists of \( k \) redundant branches, each branch containing a number of elements connected in series with branch \( i \) containing \( n_i \) elements.

The reliability of branch \( i \) is

\[
R_i = \prod_{j=1}^{n_i} P(X_{ij})
\]

Therefore the system reliability

\[
R(S) = \prod_{i=1}^{k} \left( 1 - \prod_{j=1}^{n_i} [1 - P(X_{ij})] \right)
\]
4. Models of Mixed Configurations

In this section we present two different models of mixed configurations and analyze its Reliability through combinatorial rules of probability and FTA approach

4.1 Model 1: SSP model

It represents a system which has 2 units connected in series. Each unit has two subunits also connected in series. Also each subunit has 2 subunits connected in parallel.

The reliability at stage i for a 2-2-2 element model is

\[ R_i(t) = \prod_{j=1}^{2} \left( 1 - \prod_{k=1}^{2} (1 - P_{ijk}) \right) \]

The Reliability of the system is given by

\[ R_s(t) = \prod_{i=1}^{2} \left( 1 - \prod_{j=1}^{2} \prod_{k=1}^{2} (1 - P_{ijk}) \right) \]

Thus, in general for a m-n-r model for a SSP configuration, we have

\[ R_s(t) = \prod_{i=1}^{m} \left( 1 - \prod_{j=1}^{n} \prod_{k=1}^{r} (1 - P_{ijk}) \right) \]
If the base events have uniform failure rate
\[ \lambda_{ijk} = \lambda \]
with reliability \( e^{-\lambda t} \), then the Reliability of the system is given by

\[ R_s(t) = 16e^{-4\lambda t} - 32e^{-5\lambda t} + 24e^{-6\lambda t} - 8e^{-7\lambda t} + e^{-8\lambda t} \]

The Mean time to failure is given by

\[ MTTF = \frac{0.5821}{\lambda} \]

Hazard Rate is given by

\[
Z(t) = -\frac{1}{R(t)} \left[ \frac{d}{dt}(R(t)) \right] \\
= \frac{\lambda \left( 64 - 160e^{-2\lambda t} + 144e^{-3\lambda t} - 56e^{-4\lambda t} + 8e^{-4\lambda t} \right)}{\left( 16 - 32e^{-2\lambda t} + 24e^{-3\lambda t} - 8e^{-3\lambda t} + e^{-4\lambda t} \right)}
\]

Model 2: PPS model

![Diagram of a system with 2 units in parallel, each with two subunits in parallel, and each subunit has 2 subunits in series.](image)

It represents a system which has 2 units connected in parallel. Each unit has two subunits also connected in parallel. Also each subunit has 2 subunits connected in series. All the units involved in this system are independent and the base events has the reliability factor \( e^{-\lambda_{ijk} t} \) with constant failure rate \( \lambda_{ijk} = \lambda \). Thus for a 2-2-2 elements model the reliability of the system is given by
\[ R(t) = P(X_1 \text{or } X_2) \]
\[ = P(X_{11} \text{or } X_{12}) + P(X_{21} \text{or } X_{22}) - P(X_{11} \text{or } X_{12})P(X_{21} \text{or } X_{22}) \]
\[ = [P(X_{111})P(X_{112}) + P(X_{121})P(X_{122}) - P(X_{111})P(X_{122})P(X_{112})P(X_{121})] \]
\[ + [P(X_{211})P(X_{212}) + P(X_{221})P(X_{222}) - P(X_{211})P(X_{222})P(X_{212})P(X_{221})] \]
\[ - \left\{ [P(X_{111})P(X_{112}) + P(X_{121})P(X_{122}) - P(X_{111})P(X_{122})P(X_{112})P(X_{121})] \right\} \]

In general the reliability at stage i for a 2-2-2 element model is
\[ R_i(t) = 1 - \sum_{i=1}^{2} \left[ 1 - \sum_{j=1}^{2} \left[ 1 - \sum_{k=1}^{2} (P_{ijk}) \right] \right] \]

The Reliability of the system is given by
\[ R_s(t) = 1 - \sum_{i=1}^{2} \left[ 1 - \sum_{j=1}^{2} \left[ 1 - \sum_{k=1}^{2} (P_{ijk}) \right] \right] \]

Thus, in general for a m-n-r model for a PPS configuration, we have
\[ R_s(t) = 1 - \sum_{i=1}^{2} \left[ 1 - \sum_{j=1}^{2} \left[ 1 - \sum_{k=1}^{2} (P_{ijk}) \right] \right] \]

If the base events have uniform failure rate \( \lambda_{ijk} = \lambda \) with reliability \( e^{-\lambda t} \), then the Reliability of the system is given by
\[ R_s(t) = 4e^{-2\lambda t} - 6e^{-4\lambda t} + 4e^{-6\lambda t} - e^{-8\lambda t} \]

The Mean time to failure is given by
\[ MTTF = \frac{1.041}{\lambda} \]

Hazard Rate is given by
\[ Z(t) = -\frac{1}{R(t)} \frac{d}{dt}(R(t)) \]
\[ = \frac{\lambda (8 - 24e^{-2\lambda t} + 24e^{-4\lambda t} - 8e^{-6\lambda t})}{(4 - 6e^{-2\lambda t} + 4e^{-4\lambda t} - e^{-6\lambda t})} \]

The Table below show the values obtained for \( R(t) \) and \( Z(t) \) for the SSP, PPS Models by considering \( \lambda = 0.01 \) and varying t- values
Table 1

<table>
<thead>
<tr>
<th>time (t)</th>
<th>SSP Model</th>
<th>PPS Model</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>R(t)</td>
<td>Z(t)</td>
</tr>
<tr>
<td>2</td>
<td>0.9984</td>
<td>1.6X10⁻³</td>
</tr>
<tr>
<td>4</td>
<td>0.9939</td>
<td>3X10⁻³</td>
</tr>
<tr>
<td>6</td>
<td>0.9865</td>
<td>4.4X10⁻³</td>
</tr>
<tr>
<td>8</td>
<td>0.9766</td>
<td>5.7X10⁻³</td>
</tr>
<tr>
<td>10</td>
<td>0.9643</td>
<td>6.9X10⁻³</td>
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<td>20</td>
<td>0.8749</td>
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</tr>
<tr>
<td>50</td>
<td>0.5103</td>
<td>2.3X10⁻²</td>
</tr>
</tbody>
</table>
5. Conclusion
When a System is formed from elements and units connected in parallel, series or mixed configuration, a suitable method of calculating its Reliability becomes necessary. In this paper we present two different models namely SSP and PPS models. We have calculated its Reliability, Hazard rate and Mean time to failure and a comparison study tells us that PPS model has better Reliability, less Hazard rate and moreover its Mean time to failure is comparatively more than SSP model.

6. References
[5] Dhillon, B.S., Reliability and Quality Control: Bibliography on General and Specialized